1972

Q5 $v^2 = \omega^2(a^2 - x^2)$ (as in 1974)

$\frac{d}{dt}$ both sides.

$$\frac{dv^2}{dt} = \frac{d(\omega^2a^2 - \omega^2x^2)}{dt}$$

constants

$$\frac{dv^2}{dv}\frac{dv}{dt} = \frac{d\omega^2a^2}{dt} - \frac{d\omega^2x^2}{dt}$$

$$\Rightarrow 2v\frac{dv}{dt} = 0 - \omega^2\frac{dx^2}{dt}$$

$$\Rightarrow 2v\frac{dv}{dt} = -\omega^2\frac{dx^2}{dx}\frac{dx}{dt}$$

$$\Rightarrow 2v\frac{dv}{dt} = -\omega^2 . 2x . \frac{dx}{dt}$$

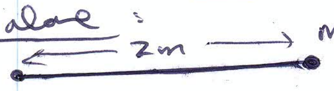
$$\Rightarrow 2v\frac{dv}{dt} = -\omega^2x\,(2v)$$

$$\Rightarrow \frac{dv}{dt} = -\omega^2x$$

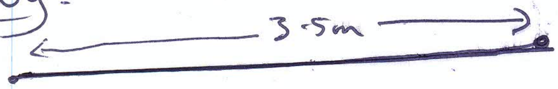
$\rightarrow$ accel $= -\omega^2x$ **qed**

which represents SHM.

String alone:



Initially:



Equilibrium position: **accel = 0**

Forces: $\leftarrow L$

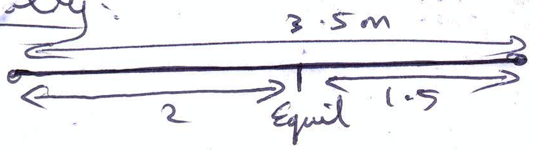
$-12(y-2)$

NII $\Rightarrow$ $-12(y-2) = 0$.

$\Rightarrow$ **$y = 2$**

Equilibrium position is 2m from point p.

Initially:



$v = 0$ when $x = 1·5$m.

$\Rightarrow A = 1·5$m

Show SHM:

Forces at point $2+x$ from ~~wall~~ point p

Forces

Hookes Law

$\Rightarrow L = k(\text{ext})$

$= k(x+2-2)$

$= 12x$.

NII $\Rightarrow$ $\Sigma \vec{F} = m\vec{a}$

$\Rightarrow -12x = 3a$.

$\Rightarrow -4x = a$

This is SHM with **$\omega = 2$**

$\Rightarrow$ Periodic Time

$T = \frac{2\pi}{\omega} \Rightarrow T = \frac{2\pi}{2} = \pi$ seconds.

Time taken to reach p

= Time taken to reach point 2m from wall [where string goes slack and so exerts no more force]

+ Time taken to travel at a constant speed the last 2m.

Time to reach 2m from wall

Start from extreme.

point 2m from wall is Equilibrium position $\Rightarrow$ Time [Extreme to mean]

$= \frac{\text{Periodic Time}}{4} = \frac{\pi}{4}$.

Speed at point of equilibrium given by $v^2 = \omega^2(A^2 - x^2)$

Here $x = 0$, $\omega = 2$ $A = 1·5$

$\Rightarrow v^2 = 2^2(1·5^2 - 0^2) = (2(1·5))^2$

$\rightarrow v = 2(1·5) = 3$.

Time to go from equil to P is

$\frac{\text{distance}}{\text{speed}} = \frac{2}{3}$

Total Time for journey $= \left(\frac{\pi}{4} + \frac{2}{3}\right)$ secs